Stochastic Scheduling

Models Real World Uncertainty

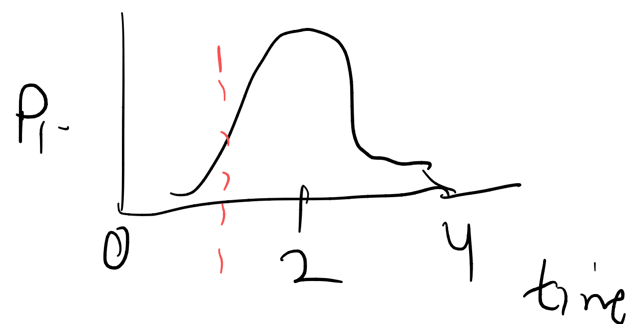
- processing times
- arrivals
- machine availability
- ...

Our Model:

- Distribution over job data known in advance.
- Realization only known when job arrives/completes or when it can be inferred.

Example:

$$p_j = \begin{cases} 1 & \Pr = 1/2 \\ 3 & \Pr = 1/2 \end{cases}$$



After 1 unit of time, if the job doesn't complete, we know that it will take 3 units.

Example

$$p_1 = \begin{cases} 1 & \Pr = 1/2 \\ 9 & \Pr = 1/2 \end{cases} \qquad E(p_1) = \tfrac{1}{2}(1+9) = 5$$

$$p_2 = \begin{cases} 4 & \Pr = 1/4 \\ 6 & \Pr = 1/2 \\ 8 & \Pr = 1/4 \end{cases} \qquad E[p_2] = \tfrac{1}{4}(4) + \tfrac{1}{2}(6) + \tfrac{1}{4}(8) = 6$$

16

$$\boxed{E[\Sigma C_j] = \Sigma E[C_j]}$$

$1 || E[\Sigma C_j]$

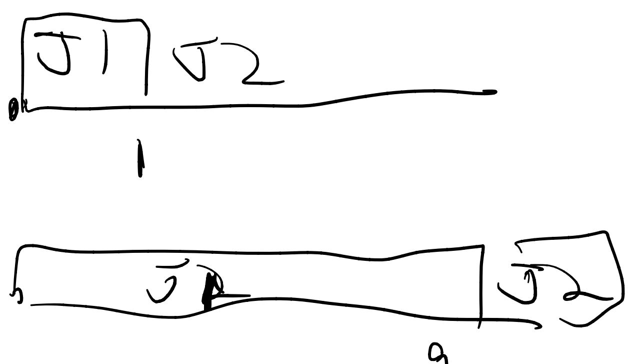
Problem: $1||\sum C_j$

Question: **What is the right algorithm? Is there still a simple ordering rule**

SEPT (expected) is the right alg.

$$E[\Sigma C_j] = \tfrac{1}{8}(1+5) + \tfrac{1}{8}(9+13) + \tfrac{1}{4}(1+7) + \tfrac{1}{4}(9+15) + \tfrac{1}{8}(1+9) + \tfrac{1}{8}(9+17)$$

$$= 16$$

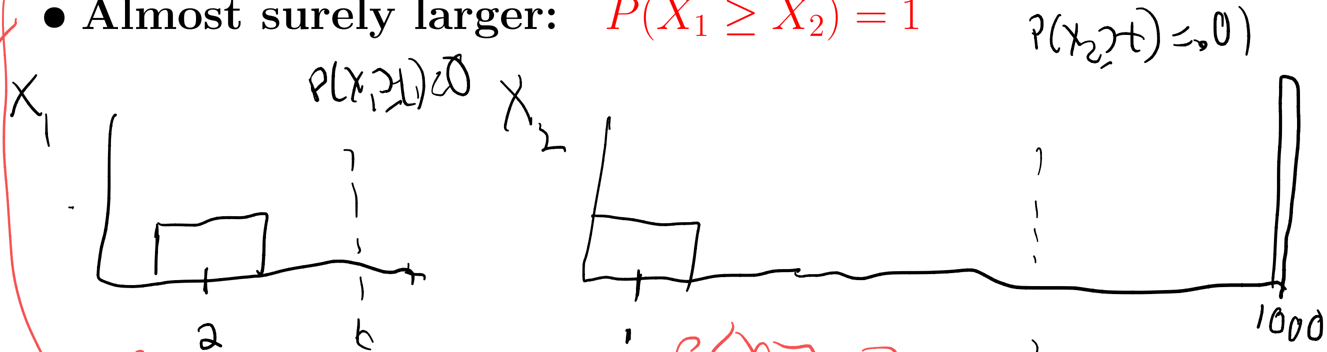


Comparing random variables

- **Density Function:** $f(x)$
- **Distribution Function:** $F(x) = P(X \le t) = \int_0^t f(x)dx$

Definitions of $X_1 \succeq X_2$

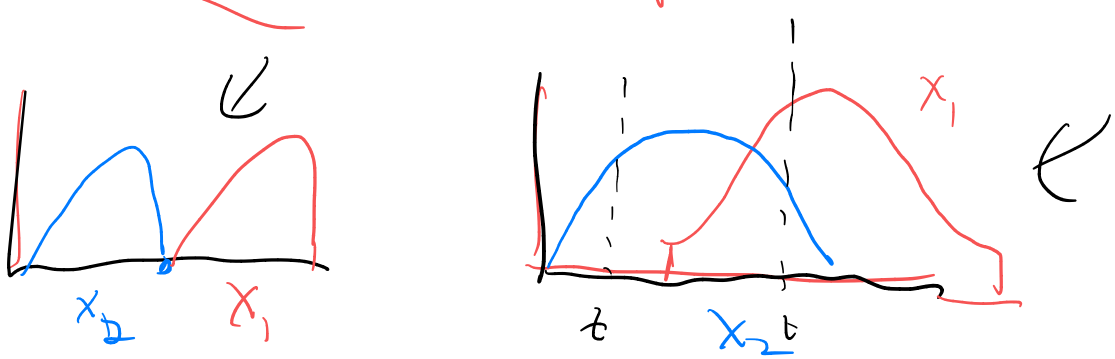
- **Larger in Expectation:** $E(X_1) \ge E(X_2)$
- **Stochastically larger:** $\forall t : P(X_1 > t) \ge P(X_2 > t)$
- **Almost surely larger:** $P(X_1 \ge X_2) = 1$



$E(X_1) = 2$

$E(X_2) = 2.01$

hard to compare



Another example, $P||C_{\max}$

$n=2$

$m=2$

$E[p_1]=E[p_2]=1$

Use $E(p_i)$

$C_1=C_2=1$

$E(C_{max})=1$

Case 1: $p_1 = p_2 = 1$

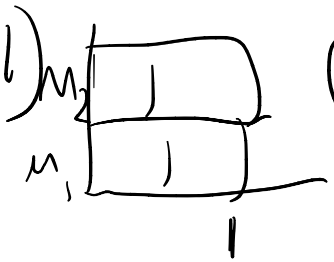
Case 2: $p_1 = 1$

$$p_2 = \begin{cases} 0 & \Pr = 1/2 \\ 2 & \Pr = 1/2 \end{cases}$$

Case 3:

$$p_1 = p_2 = \begin{cases} 0 & \Pr = 1/2 \\ 2 & \Pr = 1/2 \end{cases}$$

Case 4: p_1, p_2 **both uniform in** $[0, 2]$.

1) 

$C_{max} = 1$)

Another example, $P||C_{\max}$

$C_{\max} = \max(p_1, p_2)$

$E[C_{\max}] = E[\max(p_1, p_2)]$

$\neq \max(E(p_1), E(p_2))$

Case 1: $p_1 = p_2 = 1$

Case 2: $p_1 = 1$

$$p_2 = \begin{cases} 0 & \Pr = 1/2 \\ 2 & \Pr = 1/2 \end{cases}$$

Case 3:

$$p_1 = p_2 = \begin{cases} 0 & \Pr = 1/2 \\ 2 & \Pr = 1/2 \end{cases}$$

Case 4: p_1, p_2 **both uniform in** $[0, 2]$.

Case 3

	0,0	0,2	2,0	2,2
$C_{\max} =$	0	2	2	2

$E[C_{\max}] = \frac{1}{4}(0) + \frac{1}{4}(2) + \frac{1}{4}(2) + \frac{1}{4}(2) = \frac{3}{2}$

Another example, $P||C_{\max}$

Case 1: $p_1 = p_2 = 1$

Case 2: $p_1 = 1$

$$p_2 = \begin{cases} 0 & \Pr = 1/2 \\ 2 & \Pr = 1/2 \end{cases}$$

Case 3:

$$p_1 = p_2 = \begin{cases} 0 & \Pr = 1/2 \\ 2 & \Pr = 1/2 \end{cases}$$

Case 4: p_1, p_2 **both uniform in** $[0, 2]$.

Case 2 $\quad C_{max}$

$\frac{1}{2}$ $\quad$ 0, 1 $\quad$ 1

$\frac{1}{2}$ $\quad$ 1, 2 $\quad$ 2

$$\frac{1}{2}(1) + \frac{1}{2}(2) = \frac{3}{2}$$

Another example, $P||C_{\max}$

Case 1: $p_1 = p_2 = 1$

Case 2: $p_1 = 1$

$$p_2 = \begin{cases} 0 & \Pr = 1/2 \\ 2 & \Pr = 1/2 \end{cases}$$

Case 3:

$$p_1 = p_2 = \begin{cases} 0 & \Pr = 1/2 \\ 2 & \Pr = 1/2 \end{cases}$$

Case 4: p_1, p_2 **both uniform in** $[0, 2]$.

$$E\left[\max(p_1, p_2)\right] = \frac{4}{3}$$

$[0,2]$

$\boxed{Z = \max(Y_1, Y_2)}$

$E[\max \phi_j]$

Two R.V. Y_1, Y_2 chosen from $[0,2]$

$$E[\max(Y_1, Y_2)] = \int_0^2 t \cdot \Pr(Z = t)\, dt$$

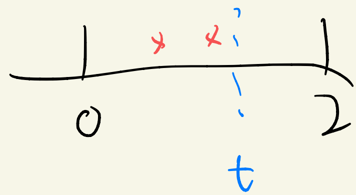
$$= \int_0^2 \left(1 - \Pr(Z \le t)\right) dt$$

$$\Pr(\max(Y_1, Y_2) \le t)$$

$$= \Pr(Y_1 \le t \wedge Y_2 \le t)$$

$$= \Pr(Y_1 \le t) \cdot \Pr(Y_2 \le t)$$

$$= \frac{t}{2} \cdot \frac{t}{2} = \frac{t^2}{4}$$



$$= \int_0^2 \left(1 - \frac{t^2}{4}\right) dt = \frac{4}{3}$$

1 hr (Wed NY times food)

CP = 95

29

1) Make Dough

Mixing, melting, knead 7

3 4 7

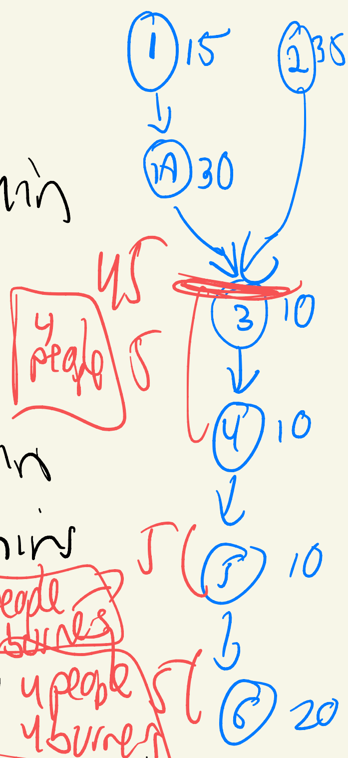
15, sit for 30 min

2) Making filling

- cutting onions & pot.
- cooking pot. 30 min
- cook onions > 10 mins

mixing

35 mins



3) Roll dough + cut circles ~10 mins

4) Assemble perogies & fold ~10 mins

5) Boil in batches of 12 ~ 5 mins/batch

6) Fry in batches of 6 ~ 5 mins/batch

Objective Values

1. $C_{\max} = 1$
2. $C_{\max} = 3/2$
3. $C_{\max} = 3/2$
4. $C_{\max} = 4/3$

Different Models of Stochastic Scheduling

Models of Knowledge

- **static: Choose order of jobs based on distribution only**
- **dynamic: Choose order of jobs based on knowledge gained when running**

Also consider Preemption vs. Non-preemption

Example:

- $1||\Sigma U_j$

$Z_j = 1 - U_j$ (entire)

- **3 jobs with same distribution:**

after j runs for 2 units, I know if $p_j = 2$ or 8.

$$p_j = \begin{cases} 2 & \Pr = 1/2 \\ 8 & \Pr = 1/2 \end{cases}$$

at time 1, I learn if $d_j = 1$

$$d_j = \begin{cases} 1 & \Pr = 1/2 \\ 5 & \Pr = 1/2 \end{cases}$$

What is the expected objective value for:

- **static non-preemptive**
- **dynamic non-preemptive**
- **dynamic preemptive**

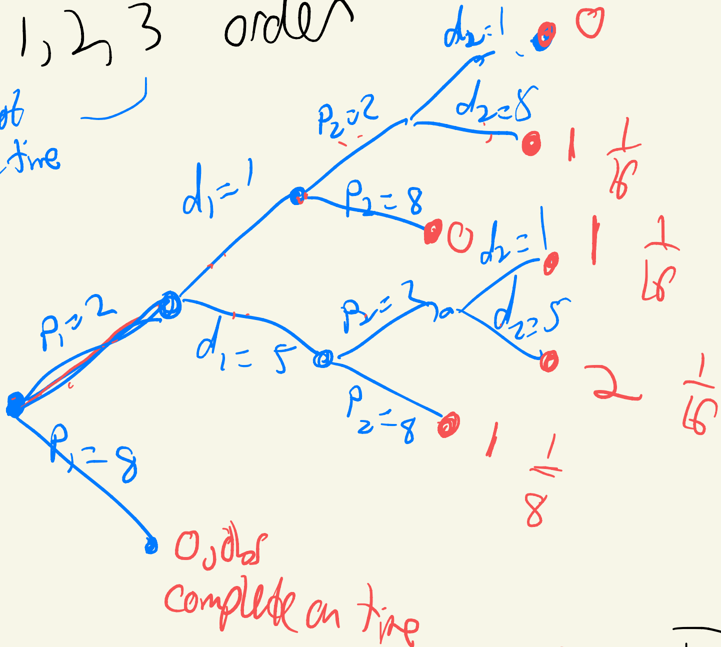
static non-preemptive

$p = (2, 8)$ $d = (1, 5)$

order jobs based on distribution info

- 1, 2, 3 order

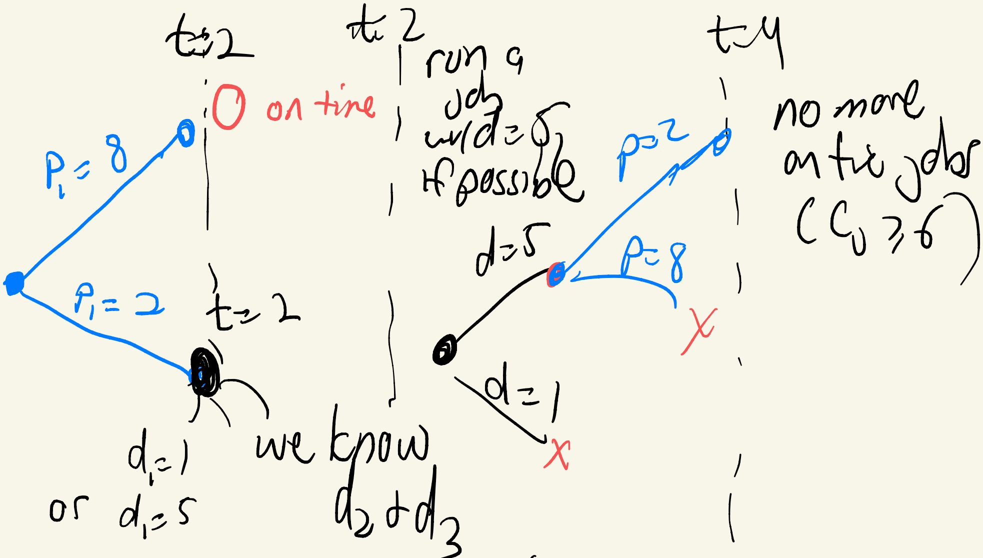
cannot be on time



$$\frac{1}{16} + \frac{1}{16} + 2 \cdot \left(\frac{1}{18}\right) + \frac{1}{8} = \boxed{\frac{3}{8}}$$

dynamic non-preemptive

(can't preempt, but we can learn)



(we don't know (p_2, p_3))

	d_2	d_3
	1	1
d_3 ⇝	1	5
d_2 ⇝	5	1
⇝	5	5

first job to run

run a job w/ d=5, if possible

$$E\left[\sum Z_j\right] = E[Z_{(1)}] + E[Z_{(2)}] + E[Z_{(3)}]$$

$$= \Pr(Z_{(1)}=1) + \Pr(Z_{(2)}=1) + \Pr(Z_{(3)}=1)$$

$$= \Pr(p_1=2)\cdot \Pr(d_1=5)$$

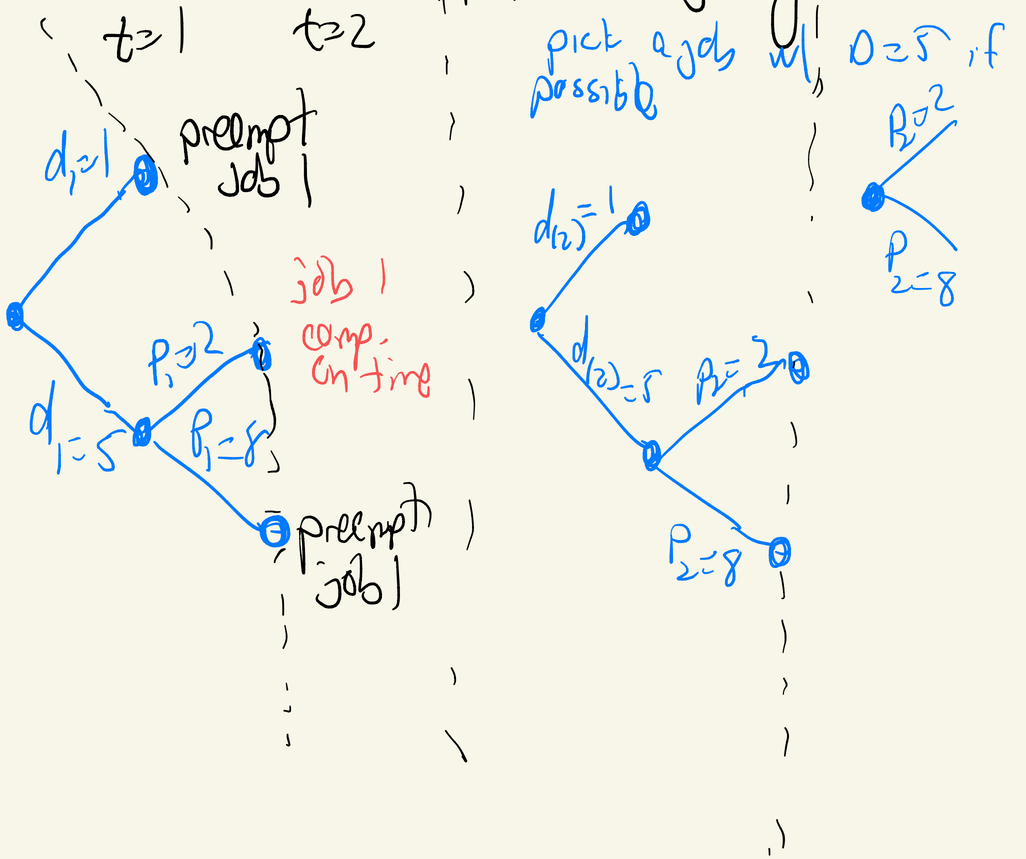
$$+ \Pr(p_1=2)\cdot \Pr(p_{(2)}=2)\cdot \Pr(d_{(2)}=5)$$

$$= \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{3}{4} = \frac{1}{4} + \frac{3}{16} = \boxed{\frac{7}{16}}$$

dynamic preemptive

t=3 or 4

t=1 or 2
job 1 not running any more



$E[Z_1] + E(Z_{(2)}) + E[Z_3]$

$= Pr(D_1=5)Pr(P_1=2) + Pr(D_{(2)}=5)\cdot Pr(P_{(2)}=2)$

$+ Pr(D_1=1)\cdot Pr(P_{(3)}=2)\cdot Pr(D_{(3)}=5)$

$= \frac{1}{2}\cdot\frac{1}{2} + \frac{3}{4}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{4} = \frac{1}{4} + \frac{3}{8} + \frac{1}{16} = \boxed{\frac{11}{16}}$

Another Example

- **Problem:** $1|\text{pmtn}|\sum C_j$
- **Jobs:**

$$p_1 = \begin{cases} 1 & \Pr = 1/2 \\ 3 & \Pr = 1/2 \end{cases}$$

$$p_2 = \begin{cases} 2 & \Pr = 1/2 \\ 4 & \Pr = 1/2 \end{cases}$$

$$p_3 = \begin{cases} 1 & \Pr = 1/2 \\ 7 & \Pr = 1/2 \end{cases}$$